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APPLICATION OF A QUASI-2D HYDRO-MORPHOLOGICAL MATHEMATICAL MODEL TO THE ARGENTINEAN PARANÁ RIVER.

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ABSTRACT

Herein a quasi-two dimensional horizontal hydro-morphological mathematical model is presented. The governing equations for the quasi-2D horizontal time-depending flow field are represented by the well-known approach of interconnected cells. New discharge laws between cells are incorporated. The model is capable of predicting temporal changes in water depth, velocity distribution, sediment transport, bed level, as well as water and suspended sediment exchange between main stream and flood plains. An application of the model in the middle reach of the Argentinean Paraná river is presented. Satisfactory results were obtained during model calibration, validation and application.

Key Words: mathematical modelling, water flow and sediment transport dynamics, riverbed aggradation/degradation processes.

1 INTRODUCTION

Mathematical models to analyze the one-dimensional morphological evolution of alluvial rivers, induced either by natural events or anthropic actions, have been commonly applied since the original work of De Vries (1957, 1965, 1969). In the last decades much effort was made in developing suitable 2D horizontal and 3D time-depending mathematical models to study riverbed changes. In fact, full 2D-H hydro-morphological models (Olesen, 1987; Spasojevic, 1988) as well as quasi-2D models based on stream tube concept (Darby et al., 1996; Lee et al., 1997) have been developed and implemented. In addition, laboratory experiments aiming to determine sediment transport direction on transverse sloping riverbeds (Sekine and Parker, 1984; Talmon, 1991; Talmon et al., 1995) and numerical analysis (Wan, 1981; Kovacs and Parker, 1988) have been also performed.

In modelling fluvial morphological processes the flow field can be represented in a quasi-2D horizontal time-depending system by means of interconnected cells (Cunge et al., 1980; Riccardi, 1992). Water flow parameters can be used to evaluate sediment transport processes in two directions and erosion/deposition patterns can be computed successively.

Herein a model development following this approach and its application to planning dredging projects in the Argentinean Paraná River are presented. The governing equations for the water flow are based on the well-known scheme of interconnected cells, with the addition of new discharge laws. For both main stream and flood plains, fluvial type links neglecting inertial terms and expressing one-dimensional discharge laws from each cell were incorporated. However, for the main stream it was necessary to take into account the influence of lateral water level gradients on the discharge from each cell. Using measured

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velocity data in different cross sections of the Paraná River the model performance was tested and a comparison with a full 2D-H hydrodynamic model was also performed.

As far as the solid phase is concerned the model can use the total load transport equation of Engelund & Hansen (1967) or the simplified bed load and suspended transport equations of Van Rijn (1984a, 1984b). Erosion/deposition rates are computed separately for bed load and suspended load by solving the corresponding sediment continuity equations in two space directions. Satisfactory results were obtained by testing the model against measured hydro-morphological data in the Paraná River.

2 GOVERNING EQUATIONS OF QUASI-2D HYDRO-MORPHOLOGICAL PROCESSES

2.1 Water flow

Considering the interconnected cells approach, continuity equation for the liquid phase can be expressed in the following way:

$$As_i \frac{\partial z_i}{\partial t} = \sum_k Q_{i,k} \tag{1}$$

where z_i is the water level in the i-th cell; As_i is the corresponding surface area of the i-th cell, t is the temporal coordinate; and $Q_{I,k}$ is the water discharge between cells i and k. Water discharges are expressed as functions of water levels: $Q_{i,k}=Q\ (z_i,\ z_k)$ and for flood plain cells they are represented by one-dimensional discharge laws of the type:

$$Q_{i,k} = F(\Delta z_{i,k}) K \left(\frac{|z_i - z_k|}{\Delta l} \right)^{1/2} \tag{2}$$

where $F(\Delta z_{i,k})=\pm 1$ depends on whether the water level difference between cells i and k is positive or negative; $K=A_t\ h^{2/3}/n$ is the conveyance factor which is function of water levels in cells i and k, with A_t being the transversal area, n being the Manning roughness coefficient and h being the water depth; Δl is distance between the points where z_i and z_k are computed. Equation (2) is derived from the 1D Saint Venant dynamic equation in which inertia terms are neglected.

Analogously, for the cells located in the main stream the discharges laws are obtained from the 2D Saint Venant dynamic equation by neglecting the inertia terms:

$$\left.\frac{\partial z}{\partial x}\right|_i + \frac{n^2 uU}{h^{4/3}} = 0 \quad , \quad \left.\frac{\partial z}{\partial y}\right|_i + \frac{n^2 vU}{h^{4/3}} = 0 \tag{3a, 3b}$$

where u is the mean flow velocity in longitudinal direction x; v is the mean flow velocity in transversal direction y; and U is the velocity vector module: $U = (u^2 + v^2)^{0.5}$. Combining equation (3a) and (3b) U can be expressed as follows:

$$U = \frac{h^{2/3}}{n} \left[\left(\left.\frac{\partial z}{\partial x}\right|_i \right)^2 + \left(\left.\frac{\partial z}{\partial y}\right|_i \right)^2 \right]^{1/4} \tag{4}$$

By substituting equation (4) into equations (3a) and (3b) the mean flow velocities u and v can be determined explicitly as functions of water level gradients in x and y directions. In this way, the discharge laws for the main stream cells takes into consideration the influence of water level gradients in both directions.

In order to deal with special features of fluvial systems, weir-like discharge laws representing natural sills, levees, roads, etc., are also included in the model. For a free overflow weir the discharge law reads:

$$Q_{i,k} = \Phi_f \left(z_i - z_w\right)^{3/2} \tag{5}$$

and for a submerged weir:

$$Q_{i,k} = \Phi_d \left(z_k - z_w\right)\left(z_i - z_w\right)^{1/2} \tag{6}$$

where z_w is the weir elevation, $\Phi_f = C_{q1}\ B\ (2g)^{0.5}$ and $\Phi_d = C_{q2}\ B\ (2g)^{0.5}$, with $C_{q1,2}$ being the discharge coefficients, g being the acceleration due to gravity and B being the weir width.

2.2 Sediment

Sediment balance in two horizontal dimensions is performed separately for bed load and suspended load. Continuity equation for bed load reads:

$$\frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} + \varphi_b = 0 \tag{7}$$

where q_{bx} and q_{by} are the volumetric bed load transport rates per unit width in x and y directions, respectively; and φ_b is the erosion/deposition rate associated to bed load. Continuity equation for suspended load without horizontal diffusion takes into account the storage of suspended sediments in the water column defined at each cell:

$$\frac{\partial (Ch)}{\partial t} + \frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} + \varphi_s = 0 \tag{8}$$

where q_{sx} and q_{sy} are the volumetric suspended load transport rates per unit width in x and y directions, respectively; φ_s is the erosion/deposition rate associated to suspended transport; and C is the mean sediment concentration. The temporal bed level changes are obtained through the summation of the corresponding erosion/deposition rates as follows:

$$(1-p)\frac{\partial z_b}{\partial t} = \varphi_b + \varphi_s \tag{9}$$

where z_b is the bed level and p is bed material porosity.

The total volumetric sediment transport rate per unit width in longitudinal direction is calculated as function of local flow and sediment parameters by means of Engelund & Hansen (1967) equation:

$$\frac{q_{tx}}{\sqrt{(s-1) g\, d_{50}^3}} = \alpha\, u^2 \frac{u_*^3}{\left[(s-1) g\, d_{50}\right]^{5/2}} \tag{10}$$

where α=0.05; s is the specific gravity of sediments; d_{50} is the grain-size of bed material for which 50% is finer; $u_*=(ghS_f)^{0.5}$ is the shear velocity, with S_f being the friction slope. Sediment transport given by equation (10) is divided to bed load and suspended load by using Van Rijn's (1984c) transport ratio T_R:

$$q_{bx} = \left(\frac{T_R}{1+T_R}\right) q_{tx} \quad , \quad q_{sx} = \left(\frac{1}{1+T_R}\right) q_{tx} \qquad \text{(11a, 11b)}$$

$$T_R = \frac{\alpha_b}{\alpha_s}\left(\frac{d_{50}}{h}\right)^{0.2} D_*^{0.6} \quad , \quad D_* = d_{50}\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \qquad \text{(12a, 12b)}$$

where D_* is the dimensionless grain-size referred to d_{50}; α_b and α_s are coefficients equals to 0.005 and 0.012, respectively; and ν is the kinematic viscosity. In addition, the model can compute directly bed load and suspended load by using the simplified sediment transport equations of Van Rijn (1984a, 1984b) from which equation (12a) is derived (Armanini and Di Silvio, 1988, Basile and Di Silvio, 1994).

Lateral bed load and suspended load fluxes are computed by means of the following equations:

$$q_{by} = \frac{q_{bx}}{\mu}\frac{\partial z_b}{\partial y} \quad , \quad q_{sy} = \varepsilon_y \frac{\partial (q_{sx}/u)}{\partial y} \qquad \text{(13a, 13b)}$$

where μ is the dynamic friction coefficient; $\varepsilon_y = \delta\, u_*\, h$ is the lateral diffusion coefficient; q_{bx} and q_{sx} are given by equations (11a) and (11b), respectively. In equation (13a) a linear relationship between lateral bed load flux and lateral bed level gradient is assumed (Darby et al., 1996) and equation (13b) takes into account the mechanism of transversal diffusion of the corresponding longitudinal suspended load transport.

3 NUMERICAL MODEL

Water flow and sediment equations are solved by means of finite difference approximation of the relevant partial differential equations. Water levels in each computational cell are determined by an implicit algorithm and water discharges are successively obtained by applying the discharge laws between cells. After that, sediment transport rates are calculated as a function of local flow and sediment parameters. Using an explicit algorithm, erosion/deposition rates of both bed load and suspended load are determined, and bed levels are consequently updated. The necessary initial conditions are represented by the water levels and discharges as well as the morphological characteristics (bed levels, grain-size of bed material, etc.) at each computational cell of the simulated domain. Boundary conditions for water flow are represented by the hydrographs at the upstream end of the reach and by stage-discharge relations at the downstream boundary. As far as the solid phase is concerned the incoming sediment transport rate at the upstream end is determined by applying the sediment transport formula. The flow field can also be schematized as stream tubes by using an internal algorithm.

4 MODEL TESTING AND APPLICATION

The model described previously was applied in the middle Paraná River, specifically in the reach shown in Figure 1, which is comprised between distances 449 Km and 455 Km measured from its outlet in the La Plata River. The goal of the model application was to study different dredging scenarios in an access channel that links a harbor facility with the main navigation canal.

Bathymetric data of the whole reach collected in 1997 in conjunction with bed material sampling, suspended sediment concentration and flow velocity measurements in four different cross sections were used to test the model. A flow rate of 17000 m^3/s was measured in that occasion. Grain size distributions of bed material are in the sand range with d_{50} varying between 0.27 and 0.32 mm and geometric standard deviation ranging from 1.46 to 1.54, thus bed material can be considered as quasi-uniform. In addition, bathymetric surveys conducted in that area by the National Harbor Construction Division in 1983 and 1985 were available. Information regarding time series of water levels collected from 1904 until nowadays in a gauging station located immediately downstream of the reach (San Martin) was also available. Moreover a stage-discharge curve permanently checked near the upstream end of the reach was also collected.

4.1 Calibration process

First, the capability of the model was quantitatively assessed by comparing the calculated flow velocity and suspended load transport (excluding wash load) distributions with the measurements performed in 1997. The reach was discretized with 884 regular cells, as shown in Figure 1, according to the bathymetry measured in 1997. A constant water discharge of 17000 m^3/s was assigned at the upstream boundary and d_{50} was set equal to 0.3 mm in each computational cell. The model was calibrated by adjusting the roughness coefficient n, the coefficient α in the total sediment transport formula and the coefficients α_b and α_s in the transport ratio. In Figure 2 a comparison between calculated velocity distributions and measured ones in the four different cross sections of the reach is presented. In Figure 2 the computations performed with a well-established full 2D-H hydrodynamic model (Riccardi, 1994) are also shown. The best agreement between calculated and measured velocity distributions was achieved by using n values varying between 0.025 and 0.032. Note that the results obtained with the present model agree very well with measurements and also note that the model is running realistically as good as a full 2D-H hydrodynamic model. Figure 3 shows the comparison between calculated and measured suspended load (excluding wash load) transport rates. The best agreement during the calibration process was achieved by setting α=0.045, α_b=0.005, α_s=0.009, μ=0.7 and δ=0.13.

4.2 Validation process

The model, with the coefficients obtained in the calibration process, was validated by using the bathymetric data collected in 1983 and 1985 with the corresponding hydraulic and sedimentological data. Time series of discharge at the upstream boundary corresponding to the 1983-1985 period were derived with the aid of the recorded water levels and the stage-discharge curve. Neither velocity measurements nor grain size distributions of bed material in the simulated reach were available for that period. Grain-size d_{50} was set at 0.3 mm, that is to say, equal to that used during the calibration process. The initial bottom configuration was that of 1983 and a two years period of morphological riverbed evolution was simulated. The flow field was schematized by means of stream tubes and the morphological evolution in the stream tubes near the right bank (where the access channel is located) was analyzed. A space step of 150 m was used to discretize each stream tube. In Figures 4 (a) and (b) the calculated final bottom configurations and the measured ones for 1985, corresponding to stream tubes 1 and 3 are presented respectively. In those Figures the initial bottom configuration of 1983 is also shown. Note the important sedimentation process verified during that period and the quite good agreement between observed and computed values of bottom aggradation. For the entire simulated area the range of variation of relative error in terms of aggradation volumes in each cell was ± 20%. However, considering the area bounded by the access channel this error is reduced to ± 10%.

4.3 Model application

The morphological response of the access channel was analyzed by considering the bottom configuration measured in 1997, without dredging works and successively by assuming an initial dredging depth suitable to guarantee a water depth of 8 m with respect to a reference level RL= + 2.50 m above local zero level. This assumption leads to an initial bed level along the access channel equal to –5.50 m with respect to the local zero level. The reference level was derived from statistical analysis of a daily-recorded water level subseries, extracted from water level duration curves by considering water levels that were surpassed during 80% of the year. A one-way channel of 100 m width was considered. Under this hypothetical dredging condition the initial volume of sediments that must be removed was 300000 m^3 approximately. In both situations (without and with dredging) three different water and sediment input scenarios were analyzed. Particularly, the morphological response of the access channel was analyzed considering maximum, typical and critical discharge years.

The maximum discharge year was 1983 because its flow duration curve showed the highest values of discharge associated to the greatest duration. In fact, in that year the discharge varied between 29000 m^3/s and 27000 m^3/s during seven month approximately. The typical year was defined as the hydrograph that presented the lowest overall deviation with respect to the arithmetic mean hydrograph of the 1967-1999 series. As a result the hydrograph of 1994 was selected to represent a typical year. The critical discharge year was 1975 because its flow duration curve showed the lowest values of discharge associated to the shortest duration.

Considering the situation without dredging works the annual volumes of sediment deposed in the access channel were estimated in 180000, 87500 and 71600 m^3 for the maximum, typical and critical discharge years, respectively. On the other hand, under the hypothetical dredging condition described previously the annual sedimentation volumes were 441000, 189000 and 135000 m^3 for the maximum, typical and critical discharge years, respectively. As expected, the greatest morphological changes were associated to the highest discharge scenarios. In both situations (without and with dredging) these volumes represent an estimation of the total amount of sediment that should be removed from the access channel in order to recover the initial bed configuration condition.

The area with the highest sedimentation rates was located approximately 700 m upstream of the harbor (452.7 Km). Considering the typical discharge year, in Figures 5 (a) and (b) the temporal evolutions of bed level, water level and water depth in this area are presented, for the situations without and with dredging works, respectively. As far as bed level variations are concerned the main differences between both situations are observed in the first half of the year. At the end of the year similar bed levels are reached in this area. An analogous behavior is observed for the remaining discharge scenarios.

5. CONCLUSIONS

The quasi 2D-H model described herein is suitable to simulate the time depending water flow, sediment transport and erosion/deposition processes in sand bed rivers. The governing equations of the model have been somewhat simplified following the interconnected cells approach. However, the essential aspects of water flow and sediment transport dynamics in relatively straight sand bed rivers are preserved. The present quasi 2D-H model can be successfully compared with a full 2D-H hydrodynamic model. It was observed that the quasi two-dimensional model is more sensitive to non-uniform bed configurations than the complete model. In any case, the quasi two-dimensional approach is very attractive because it can be implemented in long river reaches (thousands of Kilometers) and for both short- and long-term simulations of water and sediment movements. Moreover, the representation of water and sediment exchanges between large flood plains and main stream can be included by considering a number of physical limits between them (levees, roads, etc.). Satisfactory results were obtained during model calibration, validation and application in the Paraná River.

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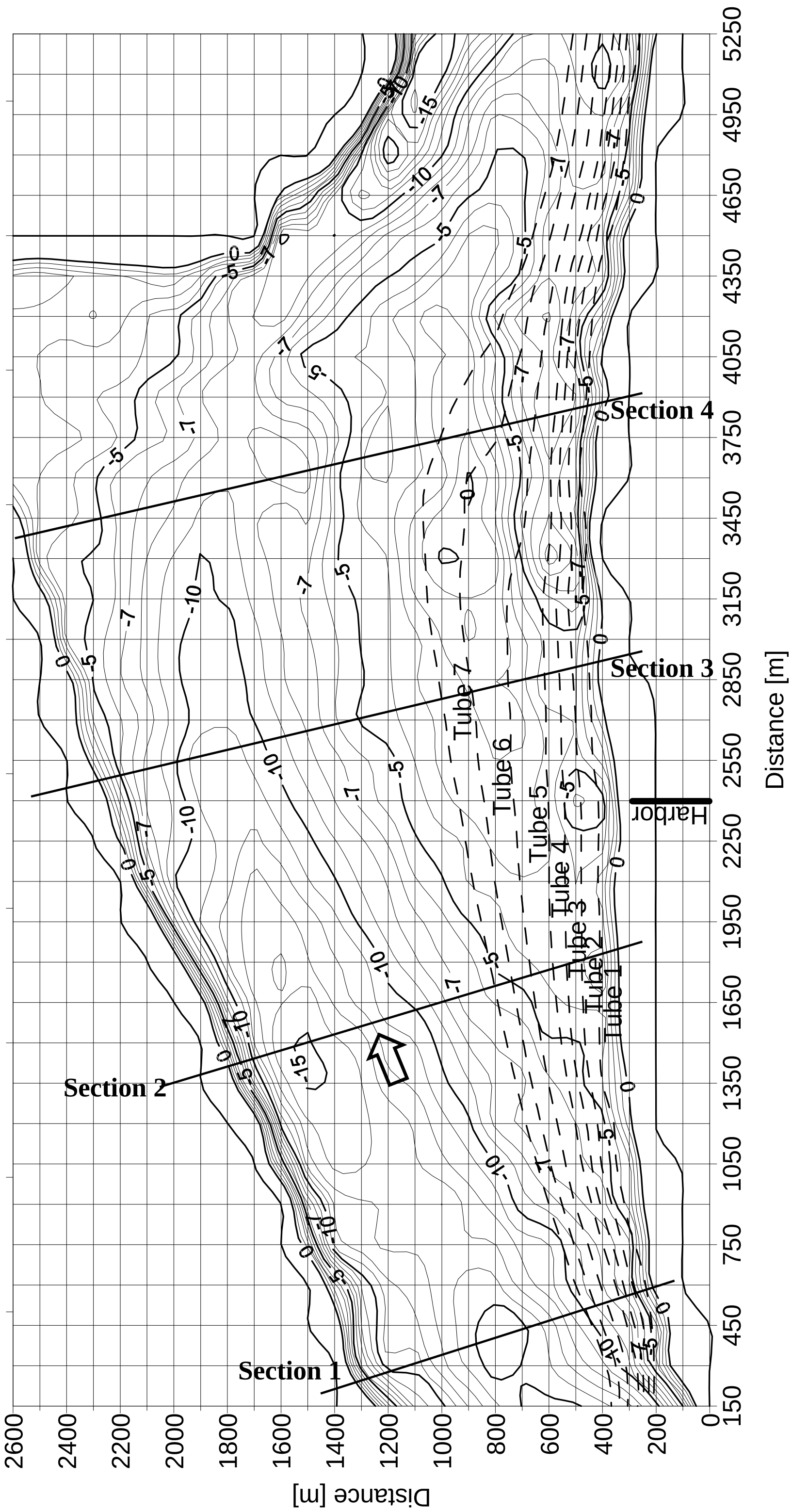


Figure 1. Bathymetry of the Paraná river reach corresponding to 1997 survey.

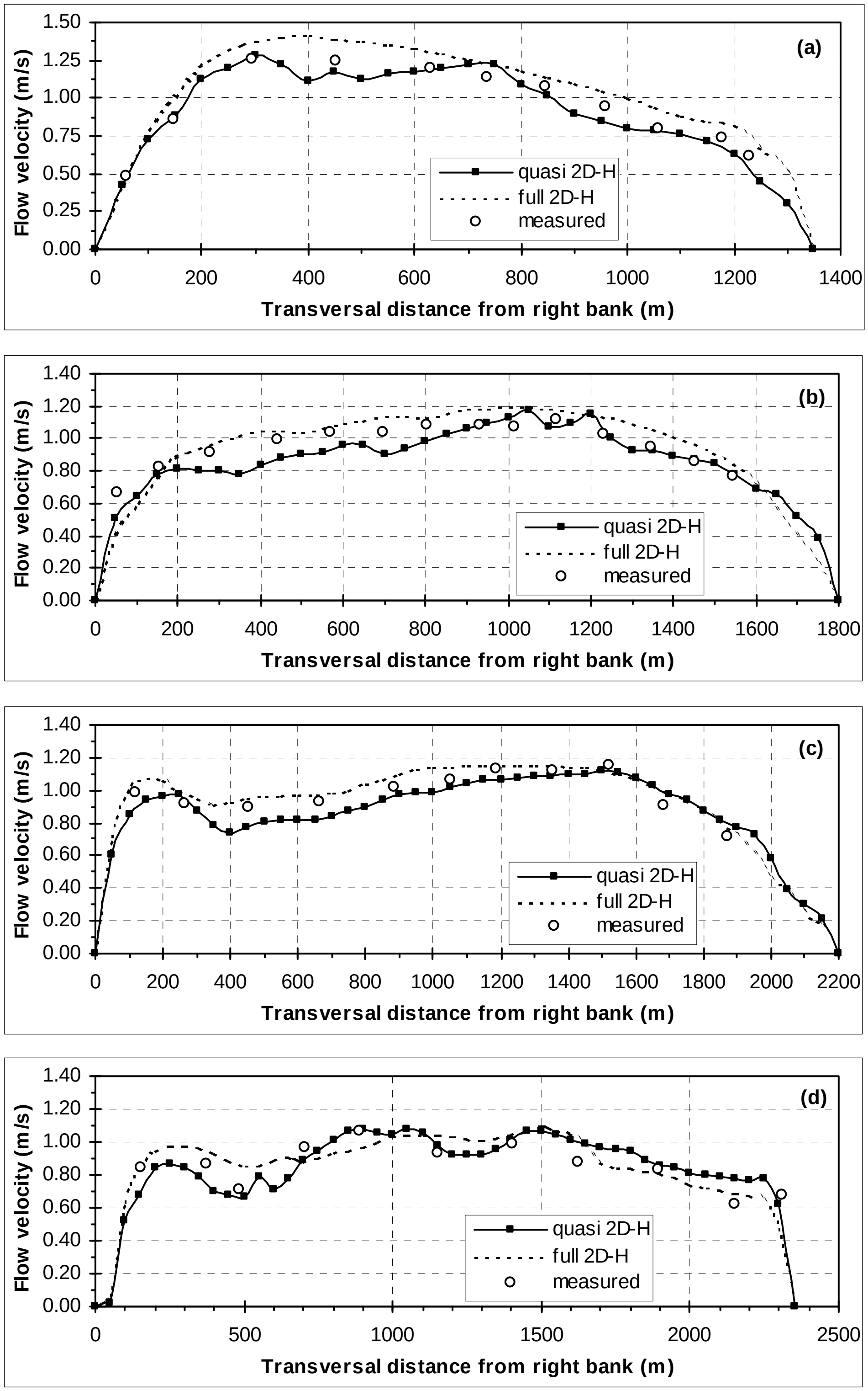


Figure 2. Comparison between calculated and measured flow velocity distributions in (a): Section 1 (453.9 Km); (b): Section 2 (452.5 Km); (c): Section 3 (451.5 Km); (d): Section 4 (450.5 Km).

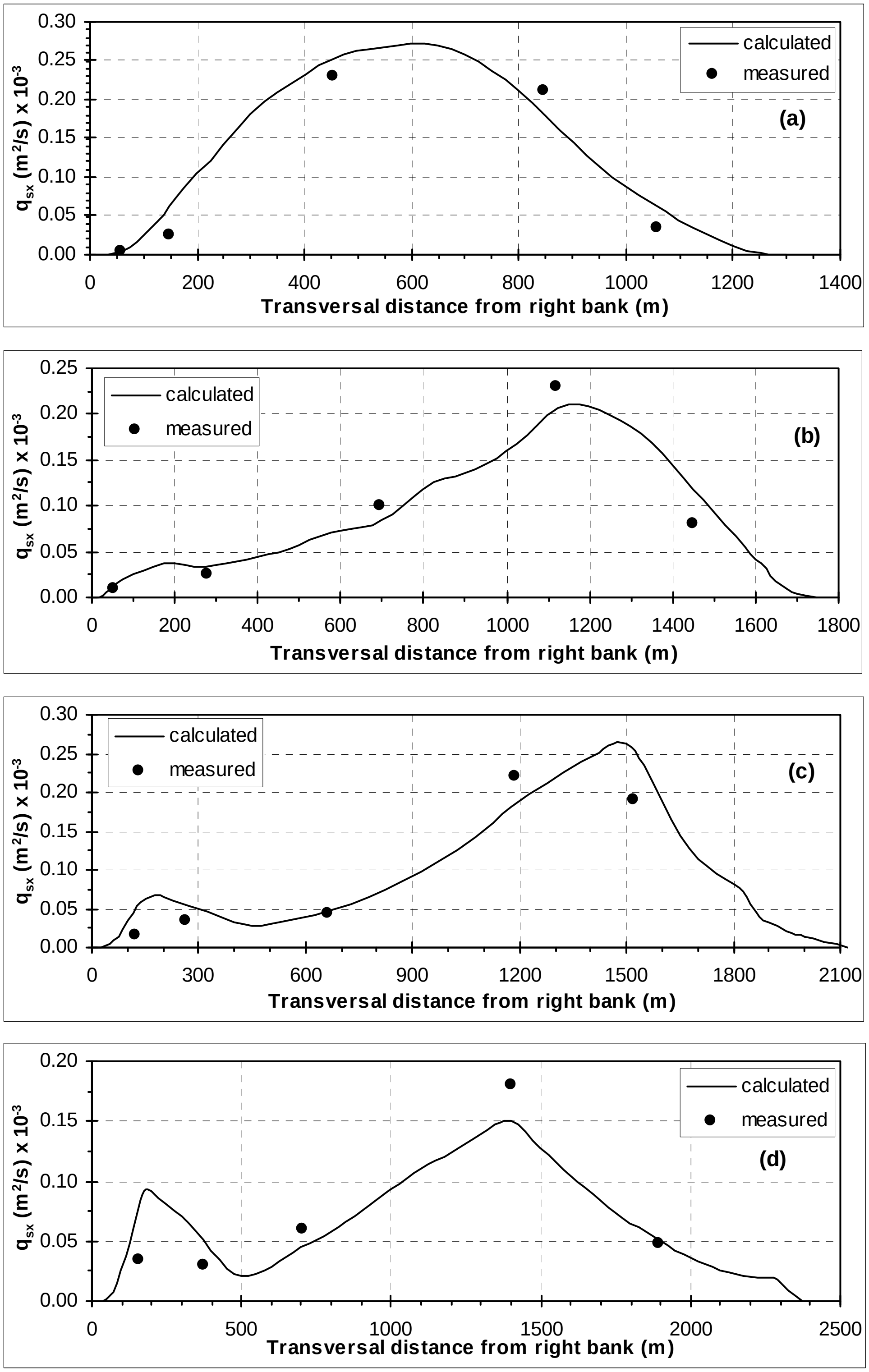


Figure 3. Comparison between calculated and measured suspended transport rates in (a): Section 1 (453.9 Km); (b): Section 2 (452.5 Km); (c): Section 3 (451.5 Km); (d): Section 4 (450.5 Km).

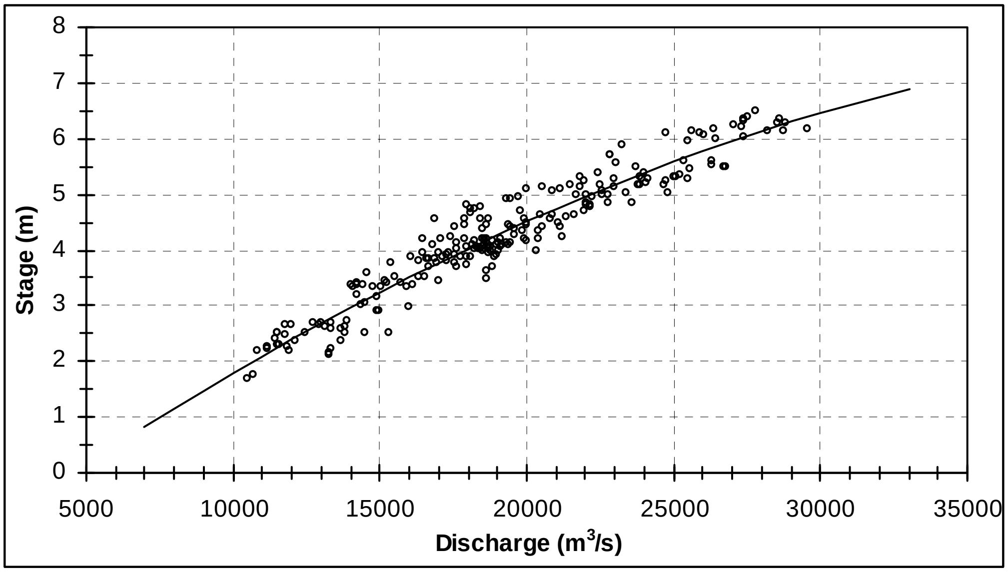


Figure 4. Stage-discharge curve of Paraná River at San Martin Harbor.

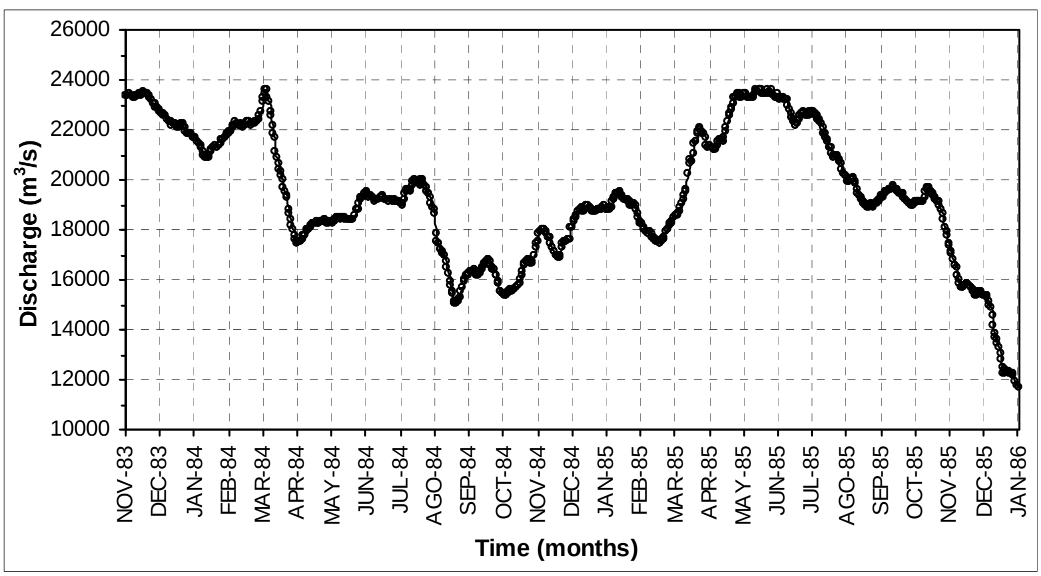


Figure 5. Input discharges corresponding to November 1983 – December 1985.

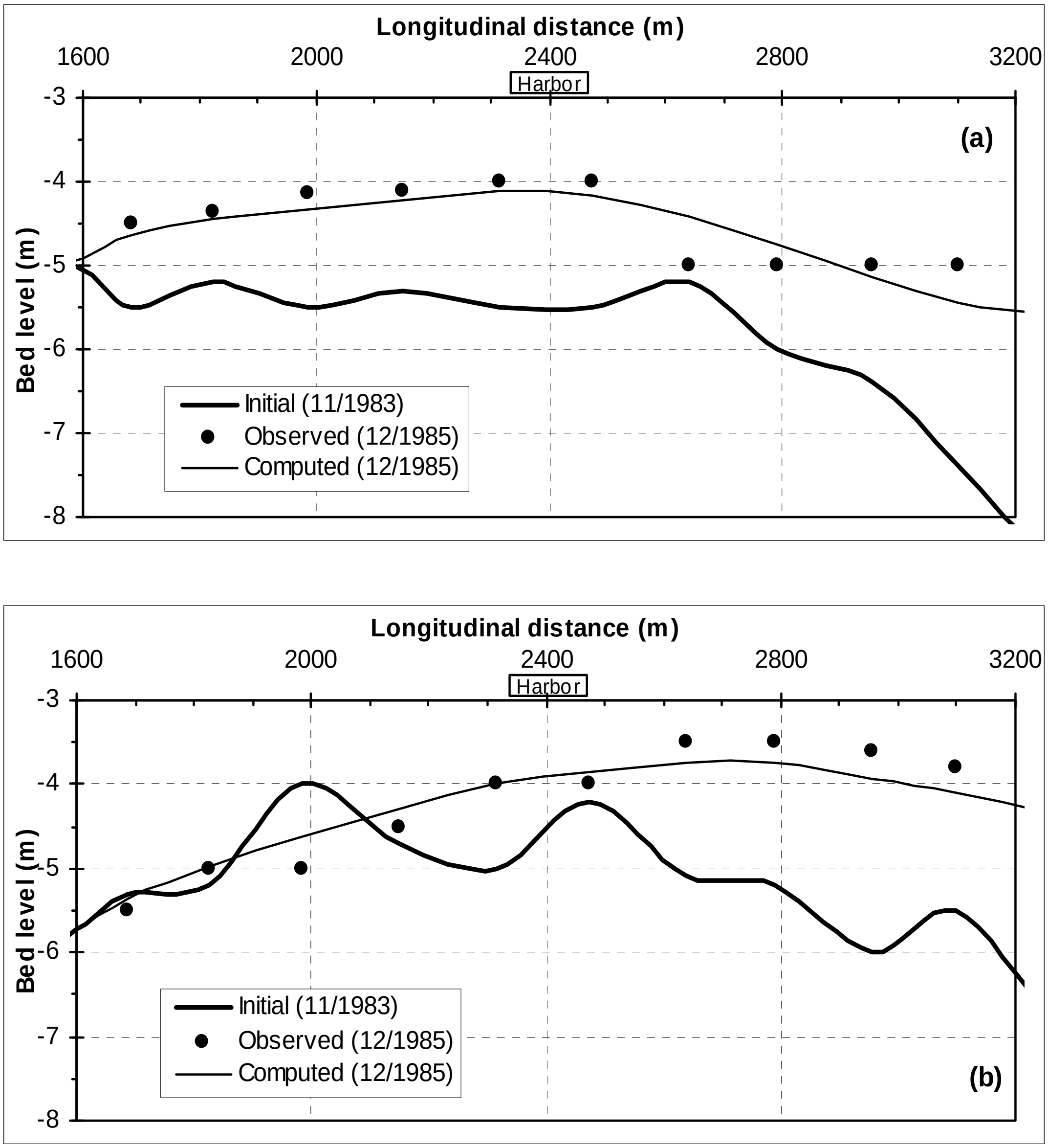


Figure 6. Comparison between observed and computed bed levels in (a): Tube 1 and (b): Tube 3.

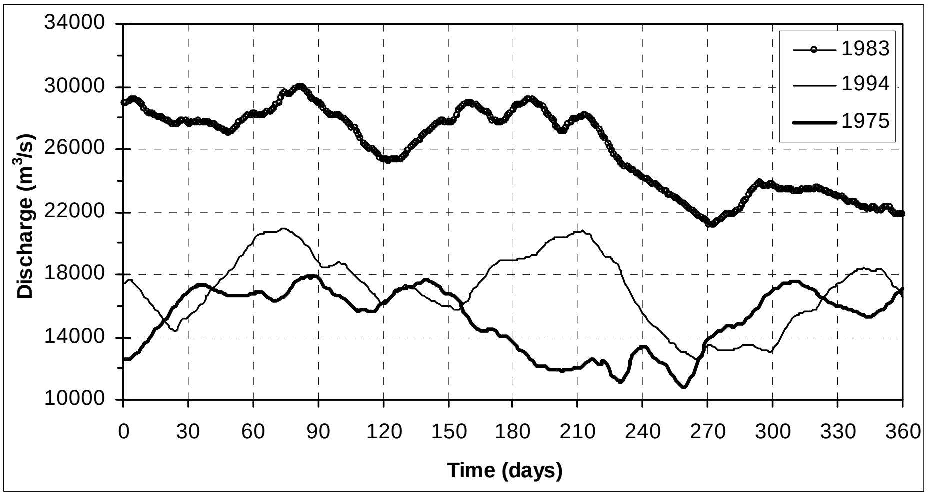
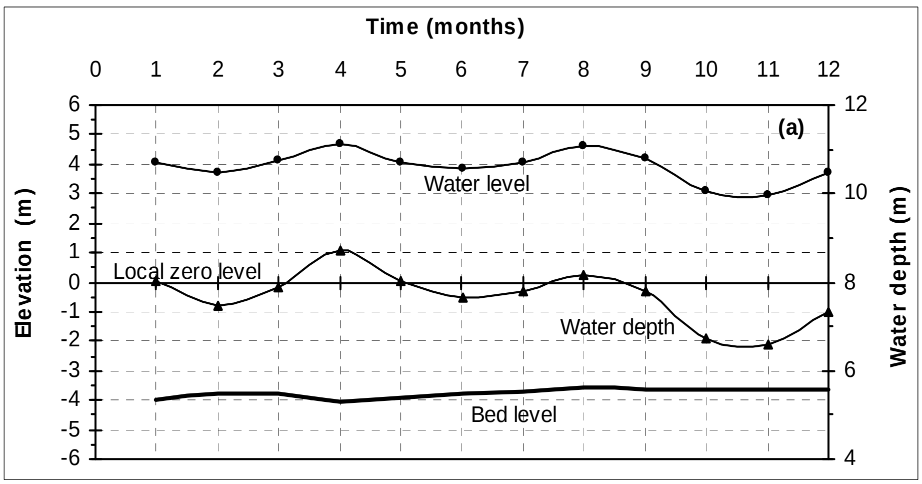


Figure 7. Input discharges corresponding to 1983 (max. year), 1994 (typical year) and 1975 (min. year).



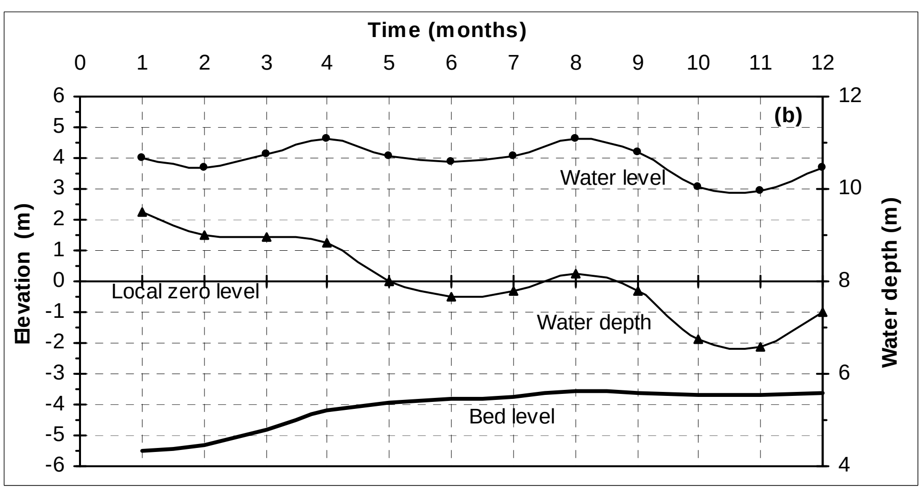


Figure 8. Temporal evolutions of water level, bed level and water depth at distance 700 m upstream of harbor location for typical discharge year considering (a): channel without dredging; (b): channel with dredging at –5.5 m with respect to local zero level.