The mathematical modelling of flood propagation for the delineation of flood risk zones

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Abstract River flood valleys are being subjected to the impact of urbanization in large cities throughout the world. Therefore, urban planning strategies based on water resources management must be established to avoid adverse environmental impacts. This paper describes a two-dimensional mathematical model for the determination of flood risk maps of the Rosario region, Argentina. The mapping was made over the River Ludueña flood plain, which embraces an area of 5000 ha, with a population of 300 000. Based on the results, regional governments are developing non-structural planning rules with the associated legislation. In this way, the hydrodynamic mathematical modelling contributes to the planning and control of water resources for a sustainable development.

INTRODUCTION

The Rosario region in the south of Santa Fe State, Argentina, is crossed by several rivers and channels. Downstream, the water courses flow in well defined channels with permanent river flow. The flood plain includes highly populated cities such as Rosario where flooding results in both economic and human losses. To solve this problem, state and local governments have built several structural works. Now, the development of non-structural design is essential to determine the legislation that allows both flood plain occupation and appropriate use. Flood risk maps for given return periods have been developed based on flow simulation modelling. This paper describes a cell model and its application which is suitable for in-stream and flood plain flow simulation. The model was developed to simulate flood propagation in urban areas both on streets and closed conduit networks for free surface flow or under surcharge.

MODEL FORMULATION AND GOVERNING EQUATIONS

The model hypotheses are appropriate for regional water courses, where rivers and streams consist of a main permanent bed and a temporarily occupied flood plain. This configuration allows division into a number of interconnected cells. The model uses a numeric resolution algorithm with no restrictions related to the links between the cells and provides both topological and hydraulic flexibility. The governing equations are those of continuity and discharge between linked cells.

Continuity equations

It is supposed that the whole i cell corresponds to a characteristic water level z_i, which is assumed in the cell centre (Fig. 1). It is also assumed that the water surface is horizontal between the borders of the cell and its value is z_i. The two fundamental hypotheses are:

(a) the volume V_i of water stored in cell i is directly related to its water level z_i: $V_i = V(z_i)$; and

(b) the $Q^n_{i,k}$ discharge between two adjacent cells i and k at a given time $n\Delta t$ depends on the energy levels: $z^n_i + \alpha_i V^2_i/2g$ and $z^n_k + \alpha_k V^2_k/2g$.

The forces originating in the local acceleration are neglected. It is possible to arrive at the continuity equation written under the differential form used by the model (Cunge, 1975):

$$A_{S_i} \frac{dz_i}{dt} = P_{i_{(t)}} + \sum_i^k Q^n_{i,k}(z_i, z_k) \tag{1}$$

There are as many equations (1) as there are cells i in the model and, also as many unknown water levels $z_i(t)$. The solution to this system exists and is unique if the set of initial conditions $z_i(t = 0)$ is prescribed (Cunge, 1975). Once this set is known, the functions $z_i(t)$ and $Q_{i,k}$ may be computed numerically. Also the boundary conditions varying in time must be prescribed.

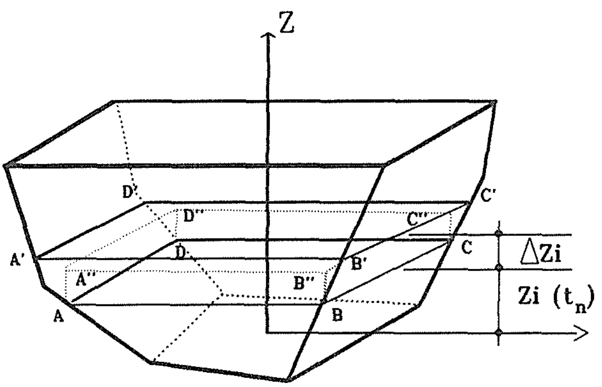

Fig. 1 Continuity equation of a cell.

Laws of discharge between the cells

River type links The $Q_{i,k}$ expression is deduced by discretization of the momentum equation for flow with negligible inertia and considering the Strickler-Manning resistance formula:

$$Q_{i,k} = \text{sign}(z_k - z_i) \frac{K}{\sqrt{\Delta x}} \sqrt{|z_k - z_i|} \tag{2}$$

where z_i and z_k are the water level in the cells; K is the conveyance coefficient, defined by $K = kAR^{2/3}$ with k: roughness coefficient of Strickler $(1/\eta)$, A: cross section and R: hydraulic radius; and Δx is the fixed distance between the cell centres.

Kinematic type links This link type is only used when the hydrodynamic information is propagated downstream. This physical case may be presented over the high cells of the modelled zone. The discharge is computed as a function of the upstream cell level:

$$Q_{i,k} = \mathrm{f}(z_k)$$

Weir type links This link type is used to represent links between cells with physical limits: cells separated by highway embankments, roads, bridges, connections between the main course and flood plain, etc. For the discharge calculation the expression of the wide crest weir is used:

$$Q_{i,k} = \mu_1 b \sqrt{2g}\,(z_k - z_w)^{3/2} \qquad \text{Free flow weir} \tag{3}$$

$$Q_{i,k} = \mu_2 b \sqrt{2g}\,(z_i - z_w)\sqrt{z_k - z_i} \qquad \text{Flooded flow weir} \tag{4}$$

where μ_1 and μ_2 are the discharge coefficients and b is the effective weir width.

Quasi-inertial river type links The deduction of the discharge between two adjacent cells is based on the complete momentum equation. Considering slow variations of z_i, z_k and $Q_{i,k}$ through time, it is possible to arrive at an explicit expression of the flow $Q_{i,k}$:

$$Q_{i,k} = \Phi_{i,k} \left\{ \mathrm{ABS} \left[\frac{z_k - z_i}{1 + \dfrac{\Phi_{i,k}^2}{2g} \left(\dfrac{1}{A_i^2} - \dfrac{1}{A_k^2} \right)} \right] \right\}^{1/2} \tag{5}$$

Energy loss type links This link is suitable for flow singularities with energy loss due to abrupt changes in the cross section. These are usually present in sewers, collector mouths, junction boxes, etc. Considering k as the portion of velocity energy loss in a link, the discharge in a contraction is defined as:

$$Q_{i,k} = \sqrt{2g} \left[\frac{z_k - z_i}{\left(\dfrac{1 + k_i}{A_i^2} - \dfrac{1}{A_k^2} \right)} \right]^{1/2} \tag{6}$$

Frictional conduit type links This link is used for connections between cells of closed conduits. The discharge is similar to the river type. The difference is given for those case of conduits under surcharge. In these situation the conveyance factor remains constant.

Transport of bed sediment and pollutants The model can simulate both the transport of bed sediment and transport of pollutants. The solid transport $G = G(Q, h, S)$ can be resolved by Engelund-Hansen, Einstein or Meyer-Peter formulations (Vannoni, 1975). The transport of pollutants is simulated by means of one-dimensional dispersion modelling (Cunge *et al.*, 1980).

NUMERICAL FORMULATION AND BOUNDARY AND INITIAL CONDITIONS

An implicit method of finite difference for the numerical resolution is used (Cunge, 1975):

$$A_{S_i} \frac{\Delta z_i}{\Delta t} = P_i + \sum_1^k Q_{i,k}^n + \sum_1^k \frac{\partial Q_{i,k}^n}{\partial z_i} \Delta z_i + \sum_1^k \frac{\partial Q_{i,k}^n}{\partial z_k} \Delta z_k \tag{7}$$

A_{si}, P_i and $Q_{i,k}$ are known in the time $t = n\Delta t$ and the increments Δz_i and Δz_k are unknown.

The model uses an algorithm based on the Gauss-Seidel method. The solid load transported, G, is resolved by uncoupled solution. The transport of pollutants is evaluated with an algorithm based on the pollutant characteristic.

There are three types of boundary conditions that the model can simulate: (a) level given as a function of time; (b) discharge given as a function of time; and (c) relationship between level and discharge. The model requires water levels in all cells to be specified at the beginning.

DESCRIPTION OF THE MODELLED PHYSICAL ENVIRONMENTS AND APPLICATIONS

The River Ludueña basin belongs to the region of Rosario city, Pampa Ondulada, Argentina. This water course drains an 800 km^2 basin, flowing toward the River Parana. The flood plain has a superficial extent of 50 km^2 over the low basin. The majority (75%) of the flood plain is rural, 15% is semi-urbanized and 10% is fully inhabited. There are around 300 000 inhabitants. The length of water courses is 19 km, with 1.2‰ slope. The overflow along the channel occurs at a discharge of 80 m^3 s^{-1}. Along a reach of 1.5 km the water course is piped in five closed conduits. These conduits have a cross section of 73.3 m^2 and they drain into the River Parana by means of an open channel flow of 0.8 km in length. The maximum discharge capacity of the conduit network is 350 m^3 s^{-1}.

The objectives of the study were to analyse the hydraulic behaviour and map the flood risk both for a natural stage (without works) and a design stage (with structural works as a retention dam upstream and new closed conduits). Maps were determined for 50, 100 and 500 years return period floods and the probable maximum flood (PMF) as a catastrophe event.

The topological and spatial discretization were satisfied with 202 cells and 311 links. Fifty-four cells correspond to open channel flow; 43 to conduit networks; 95 to flood plains of rural and semi-urbanized zones and 10 to urbanized zones. The schematization is shown in Figs 2 and 3. Topographical information was considered over a grid of points of 100 m average distance, this information was digitized for a precise determination of 0.25 m contour map. The hydraulic parameters considered in preliminary runs: roughness, efficiency coefficients, etc. were based on previous simulations (Riccardi, 1994). The model was calibrated for different floods especially with one in 1986 with a 50-year return period. Peak discharges were 500 m^3 s^{-1} for a 50-year return period,

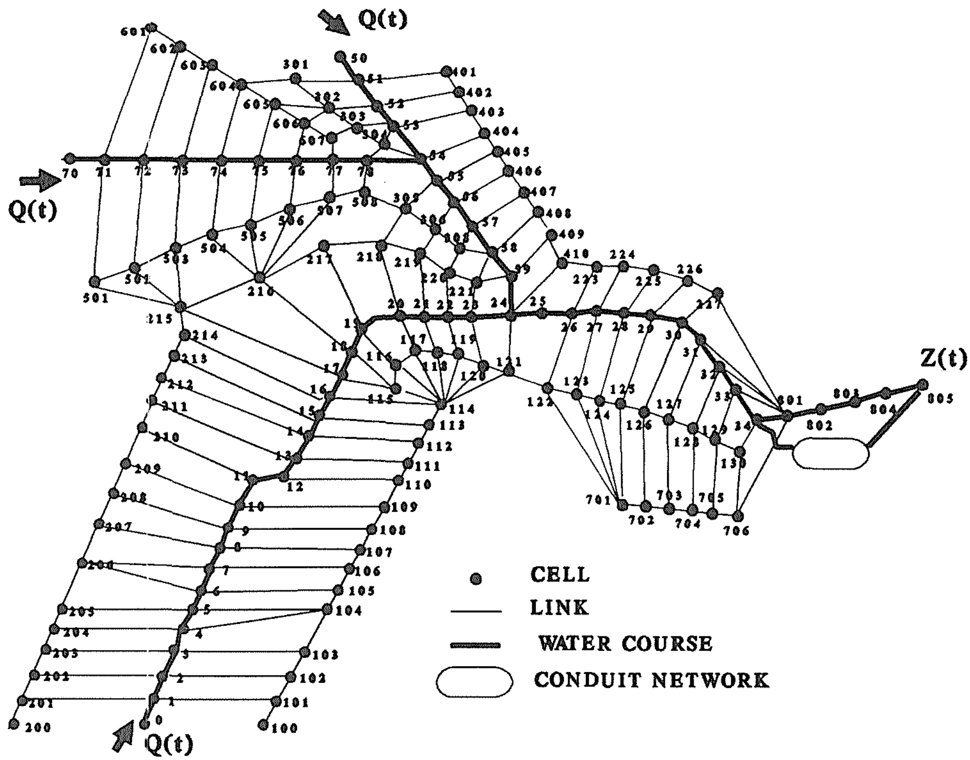

Fig. 2 Topological model representation.

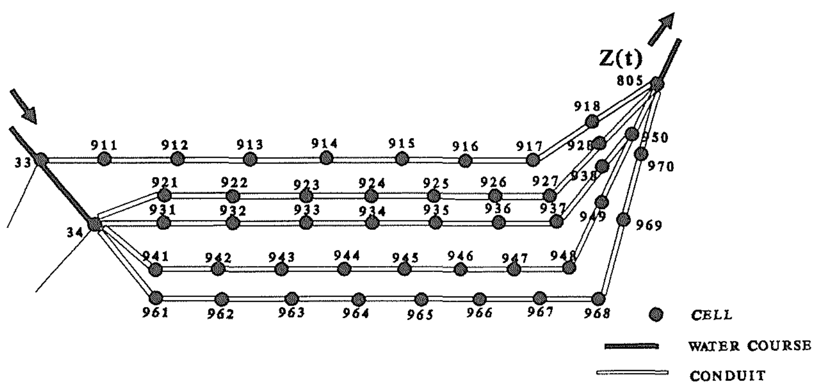

Fig. 3 Topological representation of conduit network.

700 m^3 s^{-1} for 100-year, 1000 m^3 s^{-1} for 500-year and 1700 m^3 s^{-1} for the PMF.

The results for a 50-year return period without works and 500-year return period with works are presented. Also illustrated are: the inflow from the high basin (Figs 4 and 5), the hydrographs computed at the downstream boundary over the urban zone, routing by the major system and by the conduits (Figs 6 and 7), the profiles of maximum water heights (Figs 8 and 9) and flood maps of the zone studied (Fig. 10).

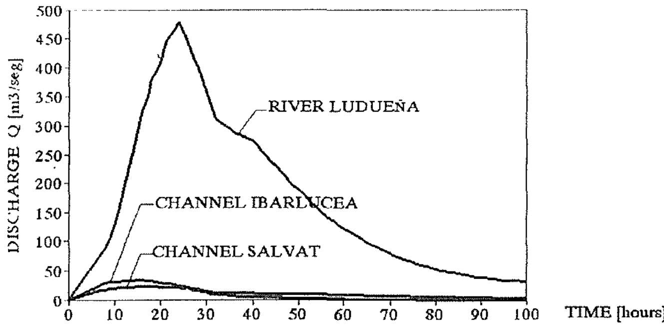

Fig. 4 Upstream hydrograph; return period 50 years, without works.

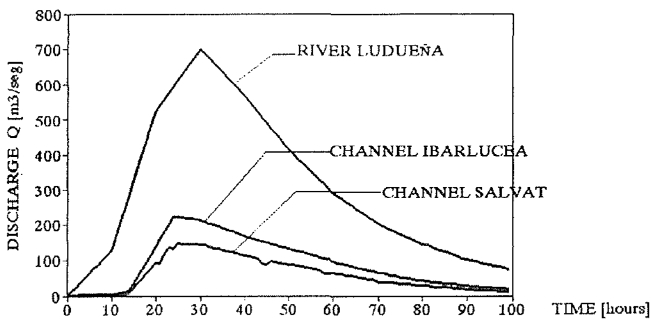

Fig. 5 Upstream hydrograph; return period 500 years, with works.

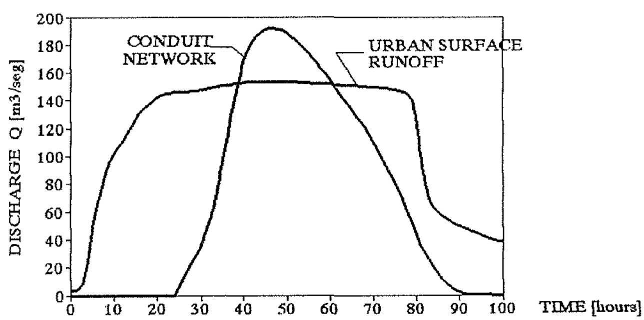

Fig. 6 Downstream hydrograph; return period 50 years, without works.

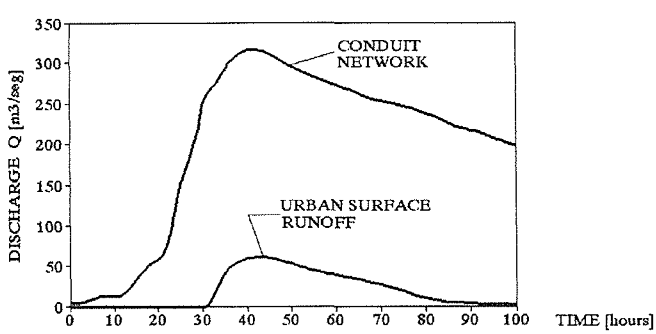

Fig. 7 Downstream hydrograph; return period 500 years, with works.

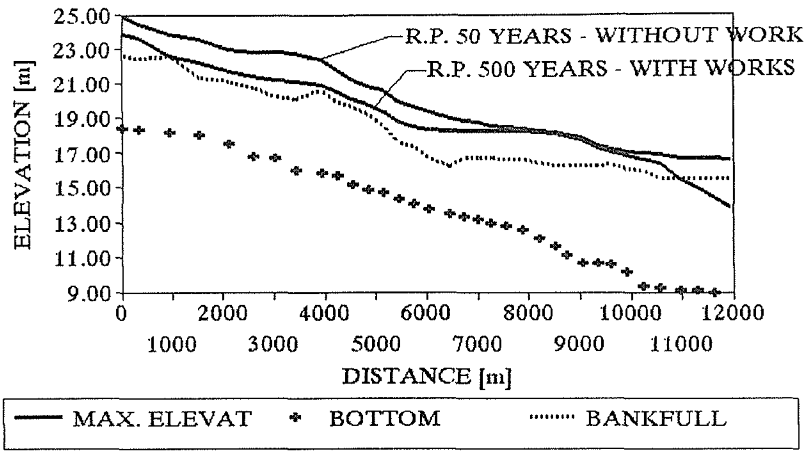

Fig. 8 Maximum elevations over river.

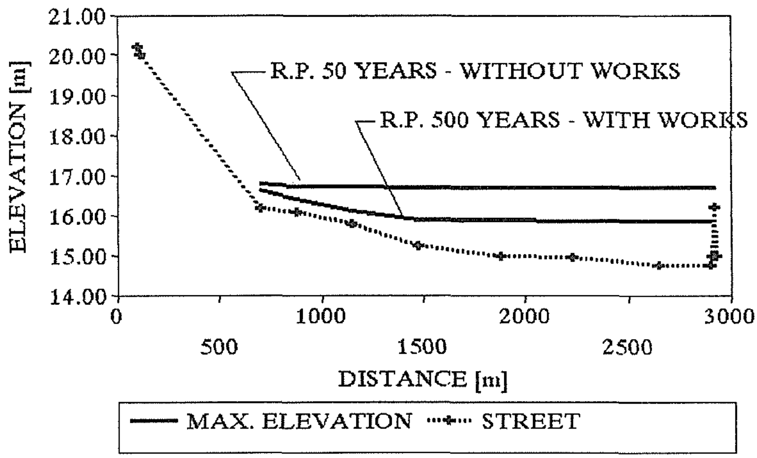

Fig. 9 Maximum elevations over urban zone.

USE OF RESULTS

Based on the flood risk area delimitation described in this paper, state and local government are planning non-structural rules and developing the associated legislation. The projected legislation proposes severe restrictions on land use and development in the 100-year flood risk zone. The map of a 500-year return flood was considered as the minimum risk zone (Fig. 11). Variable restrictions on land use development in the intermediate zone have been proposed. The rules diminish toward the limit of the 500-year flood map, according to the predicted velocity and depth maximum (Fig. 12). These maximum values ensure the safety of the inhabitants of the region.

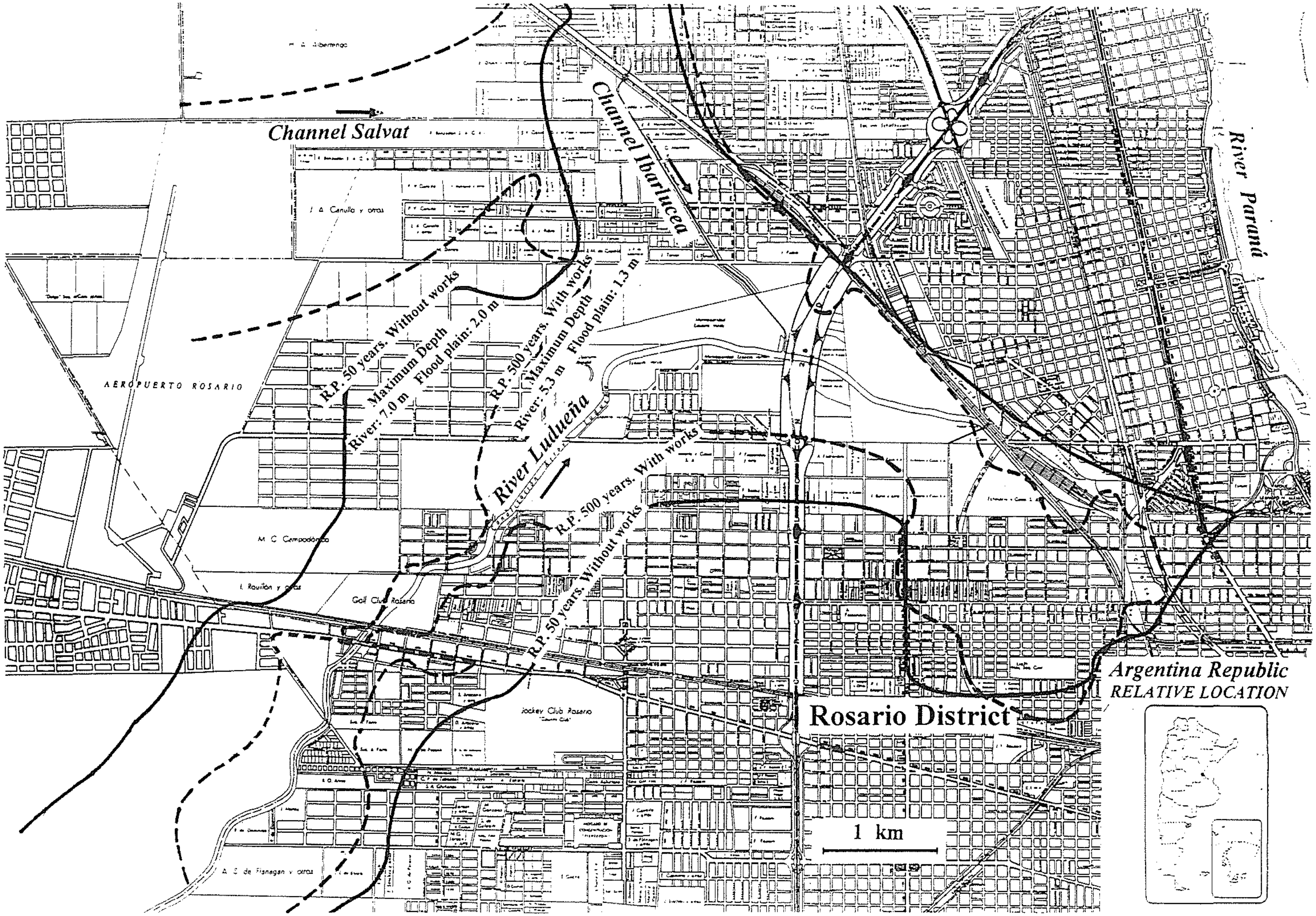

Fig. 10 Flood map River Ludueña.

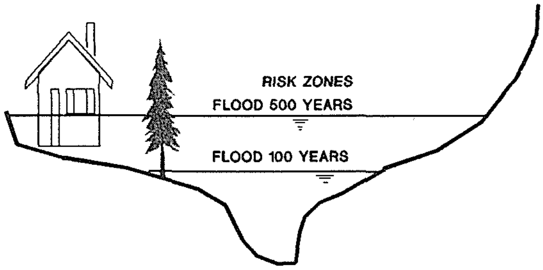

Fig. 11 Flood risk zone.

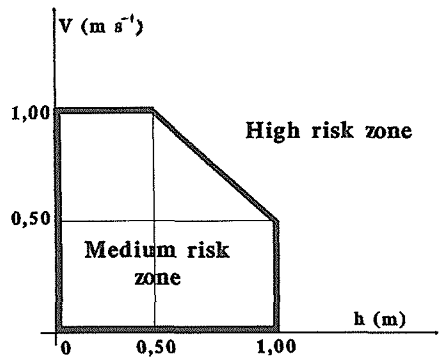

Fig. 12 Criterion for 500 years flood risk zone.

CONCLUSIONS

The model successfully simulated flow propagation by river beds, flood plains, underground conduit networks and the major system of urban zones. The applications have demonstrated a very good approximation to the observed data. The adjustment process and the first order uncertainty analysis of the variables have demonstrated the reliability and precision of the model results. It corroborated the capability of cell models to simulate different flow alternatives. By defining zones with high flood risk the model provides a tool for improved regional water resource planning which will contribute to the sustained development of urban regions.

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